XXIV. *A Difcourfe concerning the Men-*
ftrual Parallax, arifing from the mutual
Gravitation of the Earth and Moon; it's
Influence on the Obfervations of the Sun
and Planets; with a Method of obferving
it : By J. Smeaton, *F. R. S.*

Read May 12, 1768. IT is demonftrated by Sir Ifaac Newton in the *Principia,* that it is not the Earth's center, but the *common center of gravity* of the Earth and Moon, that defcribes the ecliptic ; and that the Earth and Moon revolve in fimilar ellipfes, about their common center of gravity. The fame great author has alfo inveftigated, from the different rife of the tides, when the Moon is in conjunction or oppofition to the Sun, to thofe which happen when the Moon is in her quadratures; that the quantity of matter in the Earth is to that in the Moon, as 39. 78 to 1 ; from whence, and the known diftance of the Earth and Moon, it would follow, that the common center of gravity of the two bodies falls without the furface of the Earth, by one half of its femidiameter : that is, that the center of the Earth defcribes an epicycle round the common center of gravity once a month, whofe diameter is three femidiameters of the Earth.

Dr. Gregory, in his aftronomy, has laid hold of this circumftance, in order to prove the relative gravity of the Earth and Moon, by obfervation; which is the fubject of his 60th propofition of the fourth book ;

book; in which he has demonstrated, that if an observer on the Earth makes a correct observation on the Sun's place, when the Moon is in one quadrature, it will differ from a like observation, taken in the opposite quadrature (according to a mean elliptic motion) by an angle which the diameter of this epicycle will subtend at the Sun. The same learned author has also shewn, in the scholium to the same proposition, that this quantity, or parallax, will be twice greater to Mars in opposition, and three times greater to Venus, in her inferior conjunction with the Sun.

The difference thus produced in the apparent place of the Sun, and of all the primary planets being governed by the Moon, and having it's period the same, may perhaps be not unaptly called the *menstrual parallax*.

Now if, with Sir Isaac Newton, the relative gravities of the Earth and Moon are taken between the proportion of 39 and 40 to one; the menstrual parallax of the sun will come out $13''$ upon the radius of the Earth's epicycle, and will affect the solar observations at the opposite quadratures, by double that quantity, viz. $26''$: in like manner, the mean distance from the Earth of Mars in opposition, being to the Sun's mean distance, as 1 to 2. 1; and the least distance of Mars from the Earth, to the Sun's mean distance, as 1 to $2\frac{1}{4}$, the menstrual parallax of Mars will affect the observation upon him in that situation, by $56''$ and $73''\frac{1}{4}$, respectively.

The mean distance of Venus from the Earth, in her inferior conjunction, being to that of the Sun as $3\frac{1}{2}$ to 1 nearly, and not very variable, on account

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of the orbit of Venus being almoſt circular; the menſtrual parallax would affect the place of Venus, in that ſituation, by a quantity not leſs than 92″; and in all other ſituations in proportion to her diſtance; which alſo holds with reſpect to all the reſt of the planets.

Theſe diſturbing quantities are by no means to be difpenſed with, in the nice and critical ſtate that aſtronomical obſervations and calculations have arrived at, in conſequence of the diſcoveries of Dr. Bradley, who may be ſaid to have given a baſis to aſtronomy; however, could we rely upon the *data*, on which Sir Iſaac's inveſtigation of the relative gravity of the Earth and Moon is founded, we ſhould have nothing to do but to apply an equation to the particular caſes, according to the diameter of the epicycle, as deduced from the relative gravity; but whoever conſiders the great obſtructions that the water of the ſea meets with in its motion to obey the influence of the Moon; the great difficulty in aſcertaining the true height of the tides, from the many diſturbing cauſes intervening; and the many uncertainties, and want of coincidence, that have attended, and muſt attend, ſuch obſervations; muſt confeſs, that this matter does not ſeem capable of ſuch a determination from that quarter, as the preſent ſtate of aſtronomy requires.

Accordingly, ſince the time of Dr. Gregory, thoſe great aſtronomers Dr. Bradley, De la Caille, and others, have applied themſelves to determine the quantity of the menſtrual parallax from ſolar obſervations: but though theſe have given cauſe to ſuppoſe that the relative gravity of the Earth and Moon are not above $\frac{2}{3}$ of the quantity deduced from the tides; yet, as the

obſervation

observation of these small angles principally depends upon the observation of the Sun's right ascension (which, depending on the measure of time, is less capable of exact observation, than if depending on divided instruments); the deductions thence drawn seem still wanting of that certainty which the subject demands; and if to this we add, from a deduction of Mr. Maskelyne, that the relative gravity of the Earth and Moon is as 76 to 1, derived from the effect that the Moon produces in the nutation of the Earth's axis; the relative gravity, and consequently the parallaxes thereon depending, will be reduced to almost one half of those resulting from Sir Isaac's determination.

It is true, that the quantity of effect of the menstrual parallaxes will not be great, if computed upon Mr. Maskelyne's induction, for as much as that the common center of gravity will be considerably within the Earth's surface; yet, even in that case, the Sun's transit over the meridian, when the Moon is in one quadrature, will differ nearly one second of time from that observed in the opposite quadrature; and though De la Caille and Mayer have formed equations depending on the Moon, to be applied to the equation of time; yet, if we are at an uncertainty, whether the *maximum* of this equation is one second, two thirds of a second, or half a second of time, each way, we are still under a material difficulty; for though these differences are so small, that it is not easy to determine them exactly from solar observations; yet, as they are capable of creating a sensible difference in these observations, they will, so long as they remain undetermined, prevent that solidity and firmness

nefs to the folar obfervations, which is the more ne-
ceffary as they are the foundation of all the reft: but
with refpect to thofe planets, that in their periods come
nearer to us than we to the Sun, the obfervations upon
them will be affected by a greater uncertainty.

The determination of the menftrual parallax is of
ftill more importance, as it is a neceffary confideration
in the determination of the Sun's parallax ; and this,
whether deduced from Mars or Venus, as I fhall
prefently fhew more particularly; but firft I muft
ftate the quantity of the menftrual parallax, accord-
ing to the beft *data* yet known, by a contrary procefs ;
and, taking the mean quantity of the Sun's parallax,
according to the determination of Mr. Short, at $8'' 8$,
and the relative gravities of the Earth and Moon,
according to Mr. Mafkelyne, as 76 to 1, and the
mean diftance of their centers equal to $60 \frac{1}{2}$ femi-
diameters ; we fhall then have the diftance of the
Earth's center from the center of gravity, at $\frac{8}{10}$ of
the Earth's femidiameter (that is, $\frac{1}{4}$ of that femi-
diameter within the Earth's furface) and the menftrual
parallax equal to $\frac{8}{10}$ of the Sun's parallax ; confe-
quently about $7''$; and the double menftrual paral-
lax, or vacillation, arifing from the whole diameter of
the epicycle, $14''$; the mean menftrual parallax of
Mars in oppofition, $29'' \frac{1}{2}$; the greateft, $38'' \frac{1}{2}$; and
that of Venus $49''$; from hence it follows, that, was
a perfon to attempt the Sun's parallax, by the diur-
nal motion of the Earth, applied as a bafis to Mars
in oppofition, as has formerly been tried ; and fhould
the Moon be at new or full at the fame time, the
change of place of the Earth's center, in its own
epicycle, would amount to an angle feen from Mars
of

of $1''.3$ nearly ; that is, in cafe the interval between the obfervations was eight hours, and Mars at his mean diftance ; but if Mars was not at his neareft diftance, this change would in the fame time amount to $1''.7$ nearly. In like manner, if a tranfit of Venus happens near the new or full Moon (as will be the cafe next year), the time of the tranfit will be affected by a change of place, fuch as the Earth's center will defcribe in its epicycle, during the time of the whole tranfit, if the beginning and end are obferved in the fame place ; or during the difference of abfolute time, at which the tranfit appears to begin or end to different obfervers in diftant meridians. Thus, when the fame obferver fees the beginning and end in the fame place, the bafe defcribed by that obferver, from the Earth's diurnal motion, muft be corrected by the fpace defcribed by the Earth's center, in the circumference of it's epicycle, during that time ; which, if it be fuppofed of feven hours, will amount to an angle of $1''.9$, feen from Venus : but, where the beginning or end is feen by different obfervers in diftant meridians, as the difference of abfolute time can hardly amount to above 15 minutes, the change of place of the Earth's center will for that time be but fmall ; however, at the rate beforementioned, it will for 15 minutes affect the parallactic angle feen from Venus, by about $\frac{7}{100}$, of a fecond ; and the parallax of the Sun, by about $\frac{1}{400}$ part of the whole : but this proportional part will remain the fame, whether the diftance of meridians be fuch as produce a greater or lefs difference of abfolute time than 15 minutes *.

* If an error of $\frac{1}{400}$ part of the whole may be fuppofed in the obfervation for determining the Sun's parallax by the tranfit

From what has been faid, I fuppofe it will appear, that the effects of the menftrual parallax are worthy of confideration; and that nothing has been yet executed, whereby it has received a determination fufficiently accurate; for, in regard to obfervations upon the Sun, the whole quantity is too fmall to be minutely obferved in right afcenfion : and with re-fpect to the application to Mars and Venus, as fuggefted by Dr. Gregory, I do not know that any thing has been done; and indeed no wonder, as the theory of the motion of Mars and Venus has not been as yet fo critically reduced to computation, as to render their parallaxes (though in themfelves much greater) deducible with equal certainty as that of the Sun.

What I therefore have now to propofe, is a method of obferving the menftrual parallaxes of Mars and Venus, without laying any undue ftrefs upon the theory of their motions.

The firft opportunity of making an obfervation for this purpofe, will be at the next oppofition of Mars ; which, according to the Nautical Almanack, will happen the 26th of October next, in the morning ; I will therefore endeavour to illuftrate this matter by taking that as an example.

The diftance of Mars from the Earth will then be fomewhat lefs than the mean diftance, that is, as 1 to 2.2 ; and confequently his double menftrual parallax, according to Mr. Mafkelyne, will be near $31''$ in the point of oppofition. Now, as the Moon

of Venus, a neglect of the menftrual parallax may make it $\frac{1}{200}$ part of the whole.

will

will be at full, not above 12 hours preceding that oppofition, the Moon will be nearly in the moft favourable fituation for the purpofe.

For this end, let an accurate obfervation be made upon the place of Mars at the follbwing times, viz. firft, near the time of the new Moon, preceding Mars's oppofition ; or more properly àt the neareft opportunity, to the time of the Moon's oppofition to Mars ; which will happen in the night, between the 12th and 13th of October : fecondly, let the place of Mars be obferved when the Moon is neareft her quartile with Mars ; that is, between the 19th and 20th of the fame month : thirdly, let an obfervation on Mars be made when the Moon is in conjunction with Mars, the neareft to his oppofition with the Sun ; that is, between the 25th and 26th of ditto: fourthly, let Mars again be obferved when the Moon has moved on to her quartile with Mars, viz. between the 31ft of October, and 1ft of November : and fifthly and laftly, let the place of Mars be obferved, when the Moon has again got to her oppofition with Mars, which happens between the 7th and 8th of November.

Now it is manifeft, that, when the Moon is in conjunction or oppofition to Mars, the center of the Earth, the center of Mars, and the common center of gravity of the Earth and Mars, will be nearly in a right line, and confequently, that an obferver will then fee Mars, in the fame place in the heavens, as if the common center of gravity was the fame as the center of the Earth ; therefore, then the place of Mars will be unaffected by a menftrual parallax; and fuch will be the firft, third, and fifth of the obfervations above propounded.

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It is equally evident, that when the Moon is in quartile with Mars, and moving towards a conjunction, an obferver, at the Earth's center, will fee Mars more backward in the ecliptic, than if feen from the common center of gravity, by $15'' \frac{1}{2}$; and that, when the Moon is in her oppofite quartile with Mars, and moving from her conjunction, that then an obferver at the Earth's center, will fee Mars advanced in his orbit more forward by $15'' \frac{1}{2}$, than if feen from the common center of gravity; and the one obfervation checqued with the other, will, according to a mean elliptic motion, differ by the quantity of $31''$; and fuch will be the fecond and fourth obfervations above propounded.

Now, from the firft, third, and fifth, obfervations, three points of Mars's orbit will be given; which, by the help of the theory of Mars's motion in an elliptic orbit, whofe aphelion, eccentricity, and nodes, are known fufficiently near for this purpofe; the intermediate places of Mars may be inferred with the requifite degree of accuracy: and particularly, as the two intermediate obfervations, viz. the fecond and fourth, will be nearly at equal intervals of time between the three others: from hence it follows, that the difference between the inferred, or computed places, at the quartiles, and the obferved places at thofe times, will be the menftrual parallax required.

It is to be noted, that the times above fpecified are the moft favourable for the obfervation; and could thofe be made uninterruptedly from weather, there would be the lefs occafion for any other: but, as much as poffible to prevent difappointments of this kind, it will be right to begin the obfervations, a month preced-
ing

ing making the proper obfervations, at the conjuncti-
ons, quartiles, and oppofitions, of the Moon with Mars,
which will be the means of fupplying fuch obferva-
tions, as may happen to prove abortive before the
oppofition of Mars, and alfo, in cafe any of the
obfervations to be made after that oppofition fhall
prove deficient, the obfervations may be carried on for
a month or competent time afterwards. As a further
fecurity againft difappointments, as well as cheque,
it will alfo be advifable to make the proper obferva-
tions, the night preceeding and fubfequent to thofe in
which the quartiles, conjunctions, &c. happen ; for,
as the quantities will not differ confiderably from thofe
obtained on the days fpecified, with proper allow-
ances they may be brought in fupport and confirma-
tion of the former.

In like manner, when Venus is moving towards her
inferior conjunction with the Sun, as will happen next
year, the fame obfervations may be made with refpect
to her; and continued for a neceffary time, to get ob-
fervations of the place of Venus ; viz. the firft, when
the Moon is in conjunction or oppofition with Venus:
à fecond, when the moon is in her quartile with
Venus : a third, in conjunction or oppofition: a fourth,
when the moon is in her oppofite quartile to the for-
mer: and a fifth, again in conjunction or oppofition:
the fame opportunity will alfo offer when Venus is
moving from her inferior conjunction with the Sun,
and becomes a morning ftar.

In regard to the obfervation of Venus, it is remarked
by aftronomers, that fhe is to be feen with a good
tranfit telefcope, when fhe is within a few degrees of
the Sun ; but, as fhe is three times nearer the Earth,
than

than the Sun's mean diſtance, when her elongation is 25° in the inferior part of her orbit, it is plain, that the neceſſary obſervations may eaſily be made, when her menſtrual parallax will be at a medium, three times greater than the Sun's; and conſequently amounting for the whole difference to 42″.

To avoid embarraſſment in deſcription, I have hitherto ſuppoſed, that all the obſervations are made in the meridian; in which caſe the right aſcenſions will be the ſame as they would appear from the center of the Earth ; and conſequently, the planet's longitudes thence deduced, nearly the ſame : but 'tis eaſy to ſee, that if the quartile obſervations are made when the planets are conſiderably to the eaſt or weſt of the meridian, and ſo choſen, that the place of the obſerver be further diſtant from the common center of gravity, than the center of the Earth is from that center, that the baſe of the obſervations will be conſiderably enlarged. Thus, in our latitude, ſuppoſing that the quartile obſervations are made four hours before and four hours after the planet paſſes the meridian, this will produce an enlargement of the baſis by one of the Earth's ſemidiameters : and as the whole baſe or diameter of the epicycle comes out, according to Mr. Maſkelyne, no more than 1.6 of the Earth's ſemidiamers; the baſe will, according to this method, come out 2.6 ; and conſequently, at the next oppoſition, the menſtrual parallax of Mars will be thereby enlarged to 50″, the greateſt to 62″ $\frac{1}{2}$, and that of Venus at a mean, to 74″ $\frac{1}{2}$.

It muſt however be acknowledged, that no kind of obſervations of the places of the planets are of equal validity with thoſe taken with the beſt inſtruments in the

the meridian ; thofe taken with micrometers perhaps
not excepted : for however accurately fmall diftances
can be meafured by the micrometer of Mr. Dollond,
yet, as thefe meafures can hardly be reduced to the
ecliptic, without having the difference of declination
or right afcenfion from other means (except two ftars
making fomewhat near a right angle with the planet
fhould appear within the field of view at once) ; and as
in all thefe cafes the rectification of the places of the
ftars themfelves ultimately depends on meridian ob-
fervations we may perhaps be allowed to fay, that
in the moft favourable cafes of the micrometer, the
determinations thence to be drawn, are not fuperior to
meridian obfervations, and in lefs favourable cafes,
muft be inferior : however, as the micrometer obfer-
vations out of the meridian give an opportunity of repe-
tition as often as we pleafe ; and the obfervations for
rectification of the ftars concerned, can be repeated
in the meridian, as often as we pleafe alfo ; it muft be
equally allowed, that when thefe kind of obfervations
are taken, not too near the horizon, when proper
ftars offer for this purpofe, and the whole fkilfully
managed ; thefe kind of obfervations fall but little
fhort of thofe taken immediately in the meridian.
I cannot therefore hefitate to recommend, that the
quartile obfervations be taken out of the meridian, as
well as in it : in the firft place, by Dollond's micro-
meter, if ftars offer in proper pofitions ; and if not,
fecondly, by taking differences of right afcenfion and
declination between the planet and the ftars, by the
common micrometer, in cafe proper ftars offer them-
felves for this pupofe: but as it frequently happens,
that no proper ftars offer themfelves to micrometers
of

of either kind; and this is ſtill more likely to happen in the obſervations of Venus, which will be chiefly in the day light; I beg leave to offer (what to me is) a new method of obſervation out of the meridian; and which, though I eſteem it not equal to micrometer obſervations of either kind, I apprehend will fall ſo little ſhort thereof, and prove ſo much ſuperior to any other method now in practice in theſe caſes, that I hope I ſhall on this occaſion be excuſed, in giving a particular deſcription thereof: but, as it is a general method of obſerving out of the meridian, I ſhall reſerve it by way of appendix.

In the next obſervation of Mars, it has been ſtated, that, in the meridian obſervations alone, the menſtrual parallax, according to the ſmalleſt eſtimation, may be expected to amount to 31ʺ in longitude; which; turned into right aſcenſion, will make about 2ʺ of time: now, if it may be allowed, that a well-prac-tiſed obſerver can take the time of a tranſit to $\frac{1}{4}$ part of a ſecond, over a ſingle wire, if he has three wires, or more, as uſual, the mean of the three ſhould be within $\frac{1}{12}$ part of a ſecond; or within $\frac{1}{24}$ part of the whole quantity in queſtion: it is however a mat-ter of chance, whether the mean of three may or may not be within $\frac{1}{3}$ part of the whole; and as equal errors may be committed in the obſervations of the tranſits of the ſtars, wherewith the right aſcenſions of the planets in queſtion are compared; which it is an equal chance, whether they tend to correct or increaſe the errors committed in the former; yet if, as has already been propoſed, the obſervations are con-tinued for two or three months, inſtead of one; and obſervations, taken the day preceding and ſubſequent

to

to the days of conjunction, quartile, and oppofition; and this as well out of the meridian as in it; we can hardly doubt but that, if the weather fhould favour, fo many cheques would be formed, that, from the next oppofition of Mars alone, the affair may be brought within a 24th part of the whole; and, if to this be added the force of fuch determinations, as may be drawn from obfervations on Venus, before and after her tranfit over the Sun next year, it can hardly be doubted, but that thofe three will bring us within a fingle fecond of a degree, fubtended from the neareft planet; and thefe conclufions will be further ftrengthened by future obfervations; as two years will fcarcely pafs without affording one or more opportunities of this kind.

As I meant not to embarrafs myfelf with exact computations, I have conftantly fuppofed the diftance of the common center of gravity from the center of the Earth, to be a fixed quantity; whereas it will vary in the fame proportion as the Moon's diftance varies; but, as this and many other *minutiæ* will properly enter the computation, when the obfervations are made, I muft beg leave to refer them to the learned in this fcience.

Aufthorpe, April 17, 1768.

J. Smeaton.